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ABSTRACT

This three-part module focuses on the principles of nuclear fusion and on the likely nature and components of a controlled-fusion power reactor. The physical conditions for a net energy release from fusion and two approaches (magnetic and inertial confinement) which are being developed to achieve this goal are described. Safety issues associated with fusion reactors are discussed, and comparisons with other power sources are compared. Problems and discussion questions are suggested for a further exploration of these subjects. Topic areas addressed in section 1 include: fusion power; fusion/fission; fusion reactions; charged-particle repulsion; physical conditions required for the fusion process; minimum ignition temperature; and the Lawson criterion. Topic areas addressed in section 2 include: magnetic confinement--magnetic field and ion density; inertial confinement; motion of charged particles in a uniform magnetic field; magnetic confinement--particle paths; the Tokamak; and fusion reactor design. Topic areas addressed in the third section on safety and pollution of fusion reactors include: tritium; neutron activation; radiation damage; accidents; comparison with fusion-reactor hazards; and problems of scale and cost; also included are projects, questions, and suggestions for further reading and research. (JN)

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INTRODUCTION TO NUCLEAR FUSION POWER AND THE DESIGN OF FUSION REACTORS

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1981

An Issue-Oriented Module

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TABLE OF CONTENTS

0.1	Introduction: preview and goals	1
0.2	Prerequisites	1
SEGMENT 1. FUNDAMENTALS OF FUSION POWER		3
1.1	Fusion power: where it works and where it doesn't	3
1.2	Fission and fusion	3
1.3	Fusion reactions. Sharing of the released energy	5
1.4	Problem set 1	8
1.5	Charged-particle repulsion	10
1.6	Physical conditions required for the fusion process	12
1.7	The minimum ignition temperature	13
1.8	The Lawson criterion	15
1.9	Problem set 2	18
SEGMENT 2. FUNDAMENTALS OF CONFINEMENT		19
2.1	Magnetic confinement: magnetic field and ion density	19
2.2	Inertial confinement	24
2.3	Motion of charged particles in a uniform magnetic field	25
2.4	Magnetic confinement: particle paths	27
2.5	The Tokamak	28
2.6	Fusion reactor design	29
Table 1--Representative Operating Characteristics for Tokamak Reactors		31

SEGMENT 3. SAFETY AND POLLUTION OF FUSION REACTORS	.33
3.1.1 Tritium	33
3.1.2 Neutron activation	34
3.1.3 Radiation damage	34
3.1.4 Accidents	34
3.1.5 Comparison with fission-reactor hazards	.35
3.2 Problems of scale and cost	.36
3.3 Problem set 3	.38
3.4 Projects, questions, and suggestions for further reading and research	.40
Table 2	.42
Table 3	.43
Figures	.44

0.1 Introduction: preview and goals.

This module focuses on the principles of nuclear fusion and on the likely nature and components of a controlled-fusion power reactor. We describe the physical conditions for a net energy release from fusion, and the two approaches (magnetic and inertial confinement) which are being developed to achieve this goal. We discuss the safety issues associated with fusion reactors, and make a brief comparison with other power sources. Problems and discussion questions are suggested for a further exploration of these subjects.

The primary aim is to provide some of the technical knowledge of the fusion process and of fusion reactors which is necessary for informed discussion and reading on this topic.

0.2 Prerequisites.

The student is expected to be familiar with the following topics on the level at which they are usually discussed in elementary physics courses:

force, Newton's laws of motion

vectors and vector components

pressure

momentum, the law of conservation of momentum

energy, the law of conservation of energy

power

temperature

the ideal gas, $pV = nRT$

the kinetic interpretation of temperature, $\frac{1}{2} \overline{mv^2} = \frac{3}{2} kT$

magnetic field and magnetic force

mass-energy equivalence, $E = mc^2$

atomic and nuclear structure, fission

The last two topics and other relevant material are discussed in the "Summary of Some Properties of Nuclei, Nuclear Radiation, and Reactors," by P. Lindenfeld, which is designed to accompany this and related modules. Sections 1 (compositions of atoms and nuclei), 2 (mass and its equivalent energy), 6 (binding energy), 7 (fusion and fission), and 9 (slowing down of charged particles) are recommended as collateral reading.

Other prerequisites are: the basic units of the SI (MKS) system, conversion of units, scientific notation (powers of ten), and significant figures.

SEGMENT 1. FUNDAMENTALS OF FUSION POWER

1.1 Fusion power: where it works and where it doesn't.

Most of the energy on earth comes from fusion. The sun's energy makes life possible, and produced the plants which formed the coal, oil, and gas which are our main fuels today. Also the sun runs on fusion power.

Can fusion be made to work right here, on earth, as a controlled source of useful energy? We think so, but we have to admit that up to now (1980) it has not been done.

If it can be done there will be some great advantages. The fuel should be relatively cheap and widely available. It will not be necessary to burn carbon-containing materials which produce a variety of pollutants, as well as CO_2 , which may produce changes in climate if its concentration in the atmosphere gets too high.

On the other hand, a fusion reactor will require a very large and expensive installation. Some radioactivity will be produced, although far less than in a fission reactor.

We will come back to some of these questions after we have discussed the principles which underlie the fusion process.

1.2 Fission and fusion.

Let us start by comparing fission and fusion.

In a fission reactor the nuclei of uranium or plutonium are bombarded by neutrons. When a neutron is absorbed the nucleus may undergo fission, i.e., it may break into two or more pieces and some additional neutrons. The neutrons which are released may cause further fissions, and in this way propagate a "chain reaction." The neutrons are not repelled by the high positive charge of the nuclei, and can approach them without hindrance.

As Einstein showed theoretically, and as has been demonstrated experimentally innumerable times, mass and energy are equivalent. A decrease

in mass (rest mass) is accompanied by a release of energy in accordance with the relation $E = mc^2$, where m is the change in rest mass, c is the speed of light, and E is the released energy.

In the fission process the total mass of the fission products and neutrons is less than the mass of the original neutron and the nucleus that absorbs it. The decrease in mass is equivalent to the energy which is released. There are many other nuclear processes that are accompanied by a liberation of energy. However, only with fission and fusion is there the possibility of producing more energy than is consumed to make the reaction possible. In other words, only for the fission and fusion reactions is there the possibility that the process, once started, can be self-sustaining like a fire.

In a nuclear fusion reaction the nuclei of light elements join together or "fuse." Many fusion reactions are possible, but the one most likely to be used in the first fusion reactors involves deuterium and tritium.

A deuterium atom is one whose nucleus is a deuteron, the heavy isotope of hydrogen, ${}^2_1\text{H}$. It occurs in all ordinary hydrogen, as one atom for about 5500 of the light isotope. A tritium atom is one whose nucleus is a triton, an even heavier isotope of hydrogen, ${}^3_1\text{H}$. Tritium is radioactive, with a half life of 12.3 years. It does not occur in nature, but can be made through nuclear reactions, most easily by bombarding lithium with neutrons.

The fusion of a deuteron and a triton results in the formation of a nucleus of ${}^4_2\text{He}$ (an α -particle) and a neutron. The mass of the fusion products (the α -particle and the neutron) is less than the mass of the reactants (the deuteron and the triton) and energy is released.

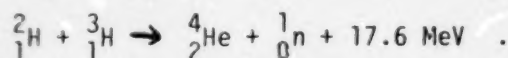
In both the fission and the fusion reactions the released energy appears as kinetic energy of the products of the reaction. As the product particles bounce around among the atoms of the surrounding material most of the neutrons

eventually cause further nuclear reactions. The charged particles, on the other hand, make collisions in which they gradually lose their kinetic energy by sharing it with the neighboring atoms. This shared random kinetic energy is "internal energy" or heat energy.

In a fusion reactor, just as in a fission reactor, or in a fossil-fuel power plant, the energy which is produced appears as heat. In order to make this energy useable and transportable it must be transformed into some other kind of energy. A number of interesting possibilities are being investigated, but today one sequence is universally used: the energy is transported to a heat engine as internal energy of a "working fluid," usually water or steam. The heat engine (usually a turbine) changes some of this energy to mechanical energy. The mechanical energy is then changed to electrical energy in an electric generator.

1.3 Fusion reactions. Sharing of the released energy.

The D-T reaction equation of a deuteron and a triton to produce an α -particle and a neutron is



The released energy (17.6 MeV) can be calculated from the masses: the mass of a deuterium atom is 2.014 u, that of the tritium atom is 3.017 u. The ${}^4\text{He}$ -atom has a mass of 4.003 u, and that of the neutron is 1.009 u. The decrease in mass is therefore 0.019 u, which is equivalent to 17.6 MeV.

(The masses are those of the neutral atoms and include one electron for each H atom and two for the He atom. Since there are two electrons on each side of the reaction equation, their masses cancel when the released energy is calculated.)

For reasons which we will discuss later, it is necessary to distinguish

between the energy carried by the charged particles and by neutrons in the reactions occurring in a thermonuclear system. In any nuclear interaction momentum must be conserved and so, assuming that the reacting nuclei are essentially at rest before the reaction takes place,

$$mv = m'v',$$

where m and m' are the masses of the particles produced in the reaction and v and v' are their respective velocities. Square both sides:

$$m^2 v^2 = m'^2 v'^2$$

or

$$m(mv^2) = m'(m'v'^2),$$

that is,

$$mq = m'q',$$

where q and q' are the kinetic energies carried by the particles of mass m and m' , respectively. If the total energy release of the nuclear reaction is Q , then:

$$\begin{aligned} Q &= q + q' \\ &= q + \frac{m}{m'} q \\ &= \left(1 + \frac{m}{m'}\right) q \\ &= \frac{m + m'}{m'} q \end{aligned}$$

$$\text{so that } q = \frac{m'}{m + m'} Q$$

$$\text{and } q' = \frac{mQ}{m + m'}.$$

In other words, the total energy is distributed between the two product particles in inverse proportion to their masses.

The amount of energy carried off by each particle in the D-T reaction is:

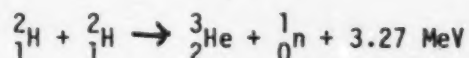
$$q_a = \frac{m_n}{m_a + m_n} Q = \frac{1.009}{4.003 + 1.009} 17.6 = 3.54 \text{ MeV}$$

and

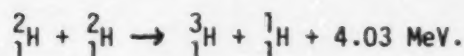
$$q_n = \frac{m_a}{m_a + m_n} Q = \frac{4.003}{4.003 + 1.009} 17.6 = 14.1 \text{ MeV}$$

We will now apply these results to some other fusion reactions.

Because of the low cost and availability of deuterium it would be preferable to use this isotope alone; the fusion process would then involve only deuterons. Two such reactions, occurring with roughly equal probabilities, are known; they are

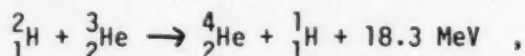


and



In the first of these two reactions, the products are a ${}^3_2\text{He}$ nucleus and a neutron with an energy release of 3.27 MeV, and in the second they are a triton and a proton, with an energy release of 4.03 MeV.

In addition to the above fusion reactions which involve only isotopes of hydrogen, there is also some interest in the nuclear fusion reaction between deuterium and ${}^3_2\text{He}$, i.e.,



since, as in the D-T reaction, there is a large energy release.

${}^3_2\text{He}$ is extremely rare in nature and there is no simple way in which it can be made, as tritium can be. Since it is a product of one of the D-D reactions this reaction could be used to provide the ${}^3_2\text{He}$ for the D- ${}^3_2\text{He}$ reaction. Both products, ${}^4_2\text{He}$ and ${}^1_1\text{H}$, carry electrical charges, and it is possible that the energy of the charged particles could be converted into electricity by a direct energy conversion process without the intermediate step of

a heat engine. This will not happen in the first generation of fusion reactors, because the minimum operating temperature for the $D-^3\text{He}$ reaction is much higher than for the $D-T$ reaction.

In view of the great abundance of the lightest isotope of hydrogen (with mass number 1), it is natural to see if nuclear fusion reactions involving this isotope could be used for the release of energy. While ordinary hydrogen, ^1_1H , is the isotope that undergoes fusion in the sun, the sun's processes take place much too slowly to be useful on earth. In the sun the generation of energy at a high rate is the result of the enormous quantity of hydrogen which is present. (It would be wise not to complain too much about the slowness of the proton-proton reaction. If its rate were higher the rate of the sun's energy production, and consequently the earth's temperature, would be correspondingly higher. The presence of life on earth as we know it would be impossible if the rate of this particular reaction were different by even a small amount.)

1.4 Problem set 1.

MKS units are generally used in the problems. Other units which are frequently used are centimeters, electron volts, and unified atomic mass units. (It will frequently be important to convert these units to the appropriate MKS units. See Table 2). Use the following information where necessary:

density of water:	10^3 Kg/m^3
density of natural lithium:	534 Kg/m^3
natural abundances:	^2H : 0.015%
	^6Li : 7.4%
	^7Li : 92.6%

The masses of the atoms of the light elements are given in Table 3. These are the masses which should be used to calculate the energy released in nuclear reactions (e.g., problems 5 to 9). To calculate the amounts of material needed for various other purposes (as in problems 1 to 4), it is sufficiently accurate to round the atomic masses off to leave one significant figure. (The atomic mass in unified atomic mass units is then numerically equal to the mass number).

1. Calculate the following for 1 m^3 of water:

- (i) the number of deuterium atoms
- (ii) the mass of deuterium
- (iii) the mass of "heavy water" (deuterium oxide, D_2O)

2. Consider a fusion power plant which produces 3000 MW ($3 \times 10^9 \text{ W}$) of heat power for one year from the D-T reaction.

- (i) What is the heat energy (in J) which is produced in one year?
- (ii) How many deuterium atoms must be used to produce this energy?
- (iii) What is the mass of D_2O which is necessary?
- (iv) What is the mass of H_2O which is necessary? What is its volume?

3. (i) What is the number of tritium atoms which is necessary for the power plant of problem 2 in one year?

(ii) What is the mass of this amount of tritium?

(iii) What is the number of ${}^6\text{Li}$ atoms which is necessary for the production of the ${}^3\text{H}$ by the reaction ${}^6\text{Li} + {}^1_0\text{n} \rightarrow {}^3\text{H} + {}^4\text{He}$

(iv) What is the number of atoms of natural lithium which is required?

(v) What is the mass of natural lithium for the year?

4. What is the mass of D_2O and of natural water which is required for the power plant of problem 2 in one year if the two D-D reactions (p. 7) are used in equal amounts?

5. Calculate from the atomic masses the released energy (Q) in MeV for each of the D-D reactions.
6. What is the fraction of the released energy which is in the form of charged-particle kinetic energy for each of the D-D reactions?
7. Calculate from the atomic masses the released energy (Q) in MeV for the D- ^3He reaction.
8. Calculate the value of Q in MeV for the reaction which produces tritium from ^6Li (see problem 3).
9. (i). Write down the nuclear reaction equation which describes the radioactive decay of ^3H by the emission of beta-rays.

(ii) Calculate the released energy from the masses. (The masses in Table 2 give the result to only one significant figure. By accident the answer is, however, accurate to three significant figures).

1.5 Charged-particle repulsion.

In contrast to the fission process, which is initiated by neutrons which are not affected by electrical forces, fusion reactions are subject to a fundamental obstacle. The nuclei are all positively charged and therefore repel each other electrically in accordance with Coulomb's law. Fusion can only occur if the nuclei come close enough for the short-range nuclear attractive forces to exceed the forces of repulsion, and this requires that the nuclei approach each other with high enough initial kinetic energy.

Two nuclei with charges Z_1e and Z_2e , respectively, have an electrostatic potential energy of $V = KZ_1Z_2e^2/r$. Here e is the magnitude of the charge on a proton or electron, r is the distance between the nuclei, and K is a constant which depends on the units which are used. In the SI (or MKS) system $K = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$.

Consider two nuclei which approach each other with a total initial kinetic energy T_1 from so far away that their initial potential energy can be taken to be zero. They will move toward each other and slow down. Their kinetic energy will decrease and their potential energy will increase, while the total energy E remains constant. This will continue until all kinetic energy has disappeared, and the energy is entirely potential with a value equal to the initial kinetic energy T_1 .

At this point the potential energy has its maximum value V_0 equal to the total energy E and therefore to the initial kinetic energy T_1 .

$$V_0 = KZ_1Z_2e^2/r_0$$

where r_0 is the closest distance of approach, under the assumption that the electrostatic repulsive force is the only force between the nuclei. (See Fig. 1). If at this point the nuclei are so close that the strong nuclear force takes over, a nuclear reaction can occur. For the light nuclei which are important for fusion the reactions can take place if r_0 is about 5×10^{-15} m. The energy E which the nuclei must acquire before they can enter into a nuclear reaction increases with the atomic numbers Z_1 and Z_2 , and even for reactions among the isotopes of hydrogen, for which $Z_1 = Z_2 = 1$, the minimum energy is about 0.29 MeV. This helps to explain the reason for choosing the isotopes of hydrogen for the fusion process. Larger energies are required for reactions involving nuclei of higher atomic number because of the increased electrostatic repulsion.

At this point it is important to note that nuclear reactions can take place at reduced but still detectable rates even when the energies are considerably less than those which we have just calculated. This behavior, which cannot be explained in terms of classical mechanics, is in accord with the theory of wave or quantum mechanics, which explains it in terms of the

so-called "tunnel effect."

1.6 Physical conditions required for the fusion process.

At very high temperatures when atoms in a gas collide with high kinetic energies, the electrons are dislodged from the atoms, leaving a mixture of unattached electrons and positively charged ions. The gas is then called a "plasma."

When a plasma is heated to still higher temperatures ions can collide at speeds sufficient to overcome the repulsive electrical forces, so that fusion can occur. A net energy release is achieved when the energy released by fusion is greater than that required to heat the plasma.

For a reactor to produce useful energy by fusion, energy has to be supplied initially in order to establish the proper conditions (temperature and ion density) for the thermonuclear reactions to take place at a significant rate. Once these conditions are reached in the plasma they must be maintained long enough to generate recoverable energy by fusion at least equal to the amount supplied initially to bring the reactor to its operating condition. If this requirement is met the system may be described as "self-sustaining." Any excess energy beyond that needed for a self-sustaining process is available for external purposes. Thus a fusion system must be at least self-sustaining for it to have any practical value. One of the conditions for a self-sustaining fusion reactor is the temperature, as will now be shown; another is related to the ion density and the confinement time.

1.7 The minimum ignition temperature.

At the very high temperatures necessary for nuclear fusion reactions a plasma loses a considerable amount of its energy in the form of radiation. This loss takes place so rapidly that the energy is not available for heating the reacting nuclei. The radiated energy is not lost entirely because it can be absorbed and used to some extent. But it escapes from the region where it is needed to maintain the fusion reaction temperature. Clearly a system cannot be self-sustaining unless the rate at which energy is produced by fusion exceeds the rate at which it is lost from the plasma as radiation. This requirement determines the minimum operating temperature for a nuclear fusion reactor.

To determine the minimum operating temperature it is necessary first to calculate the rates of thermonuclear energy production at a number of temperatures and to compare the results with the rates of energy loss as radiation. The energy gain or loss per unit time is the power, measured in watts (W). The power per unit volume is the power density. We will make the comparison in terms of the power density, measured in W/m^3 or W/cm^3 .

If Q is the average energy produced per nuclear interaction, the fusion power density P_f is given by the fusion reaction rate times Q , the energy release in one reaction. The reaction rate depends on the numbers of particles per unit volume of the two reactants, n_1 and n_2 , and on their relative velocity v . The remaining factor is the "cross section," σ , which is a measure of the probability of the reaction and has the dimensions of area. Combining these factors leads to

$$P_f = n_1 n_2 \sigma v Q$$

where the bar over σv indicates that this factor has to be averaged over all velocities.

For the D-T reaction n_1 and n_2 are the deuteron and triton particle densities. We will assume that there are equal numbers of the two kinds of particles, so that n_1 and n_2 are equal.

We will now determine the critical ignition temperature, i.e., the minimum operating temperature for the D-T reaction. We have to use the energy which remains in the system, which, in this case, is only the α -particle energy. The neutrons, since they are not subject to electrical forces, leave the point where they are produced and do not contribute to raising the temperature.

Therefore we must use 3.5 MeV for Q . The quantity $\overline{\sigma v}$ is shown, as a function of temperature, on Fig. 2. As is customary the temperature axis is not T , but the energy kT in units of keV. [The average energy per particle in an ideal monatomic gas is $(3/2) kT$]. The quantity kT is called the "kinetic temperature."

When a charged particle is accelerated by an electric or magnetic field it emits electromagnetic radiation. The photons of this radiation are called continuous x-rays, and the process is called "Bremsstrahlung."

The energy loss, per unit time and per unit volume, by Bremsstrahlung is given by

$$P_B = b n_1 n_2 \sqrt{kT}$$

where b is a proportionality constant which depends on the kinds of particles.

1.8 The Lawson criterion.

To attain a net energy release the heated fuel or plasma must be confined, without escape or contact with the material surroundings, at the necessary temperature for a length of time, τ , in seconds, and at an ion density, n , in particles per cubic centimeter, such that the product of ion density times confinement time, $n\tau$, is at least 10^{14} sec/cm³. This requirement is known as the "Lawson criterion." It describes the minimum conditions for attaining the break-even point for fusion reactions. We will now derive the relation that leads to the Lawson criterion.

The Lawson criterion describes the following situation: consider a gas heated instantaneously to a temperature T , and held there for a period of time τ . The method of confinement is not specified and the only loss mechanism is Bremsstrahlung. It is assumed that external energy is supplied not only for the initial heating but also to compensate for the radiation loss in the time τ . The plasma undergoes thermonuclear reactions during the time τ , and at the end of this period of time the confinement is removed instantly. It is now assumed that all of the energy of the neutrons, charged particles, and photons is collected and processed with some average efficiency, η , to produce useable energy. The zero-power condition is achieved when the quantity of this energy is equal to the externally supplied energy. This is the break-even point corresponding to the Lawson criterion.

We can now quantify these statements. We need to consider three amounts of energy: the amount of energy produced by the nuclear reactions, E_f ; the amount of energy to heat the plasma, E_T ; and the amount of energy which the charged particles radiate by the Bremsstrahlung process, E_B . We will use each of these quantities as an energy per unit volume.

The energy loss is $E_T + E_B$. The energy which is produced is E_f .

This energy is in the form of kinetic energy of the reaction products; i.e., in the case of the D-T reaction, of the α -particles and neutrons. A fraction η of this energy is available to supply the energy losses and to produce useable output energy.

Our assumption is that a fraction η of the heating energy E_T is recovered. The Bremsstrahlung process produces photons, and a fraction η of the photon energy is also available. Hence the condition for zero output energy is given by

$$\eta(E_f + E_T + E_B) = E_T + E_B.$$

First consider the energy (per unit volume) to heat the plasma, E_T . If the gas density is n atoms/cm³, and each atom is dissociated into an ion and an electron, the plasma will contain $2n$ particles/cm³. The plasma has a velocity and energy distribution like that of an ideal gas, so that the average energy per particle is $3/2 kT$. The energy required to raise the temperature of the gas by an amount ΔT is $(3/2 k\Delta T)(2n)$ or $3 nk\Delta T$. If the initial temperature is very small compared to the final temperature T , then

$$E_T = 3 n kT.$$

The energy lost per unit time and per unit volume by Bremsstrahlung is $P_B = bn^2\sqrt{kT}$, so that $E_B = bn^2\tau\sqrt{kT}$, where the constant b is different for each reaction.

Finally, there is the fusion energy E_f equal to $P_f\tau = n_1 n_2 \overline{\sigma v} Q\tau$ or,

$$\text{for } n_1 = n_2 = n/2, \text{ by } E_f = (n^2/4)\overline{\sigma v} Q\tau.$$

We can now substitute for E_T , E_B , and E_f in the equation

$$\eta(E_f + E_T + E_B) = E_T + E_B, \text{ and solve for } n\tau, \text{ to get}$$

$$n\tau = \frac{3kT}{\frac{\eta}{1-\eta} \frac{\overline{\sigma v} Q}{4} - b\sqrt{kT}}.$$

20

This is the general expression (in slightly different form) originally developed by Lawson as a statement of the conditions which must be satisfied in order to achieve a fusion system with zero power output.

If one plots $n\tau$ as a function of T for the D-T reaction, assuming that $\eta = 1/3$, the minimum T is about 10^8 °K and $n\tau$ is about 10^{14} sec/cm³. For the D-D reaction $n\tau$ is 100 times as large, and the temperature 5 times as high.

The two requirements which we have discussed, high temperature and adequate confinement, have been the primary focus of fusion research to date. Recently (1978) ion temperatures corresponding to $kT_i = 6.5$ keV ($T = 7 \times 10^7$ °K) have been achieved in the Princeton Large Torus (PLT) fusion facility, with a confinement time, $\tau = 0.1$ s. A value of $n\tau = 3 \times 10^{13}$ sec/cm³ has been achieved in the MIT Alcator device. These values are approaching the minimum condition needed for a fusion reactor. The next large fusion experiment, the Tokamak Fusion Test Reactor (TFTR), now under construction at Princeton, is expected to reach the break-even point. (See Fig. 4).

1.9 Problem set 2

1. Two nuclei with atomic numbers Z_1 and Z_2 approach each other. Their combined initial kinetic energies are equal to E MeV. Their distance of closest approach (assuming only electrostatic forces) is $r_0 = 5 \times 10^{-15}$ m.

Show that $E = 0.29 Z_1 Z_2$ MeV.

2. A proton whose initial kinetic energy is 5 MeV moves toward a stationary alpha-particle. What is their closest distance of approach?

3. (i) Calculate kT in eV for room temperature (300 K).

(ii) Calculate T for $kT = 1$ eV, 1 KeV, and 1 MeV.

(iii) What are the values of kT (in eV) corresponding to the temperature at the sun's surface (5000 K) and in the sun's interior (2×10^7 K)?

4. Fill in the missing steps to arrive at the equation for n_T on page 17.

5. Determine the minimum values for T and n_T for the D-T and the D-D

reactions. Use $\overline{\sigma v}$ from Fig. 2, $b_{DT} = 2.14 \times 10^{-30}$, and $b_{DD} = 5.35 \times 10^{-31}$.

SEGMENT 2. FUNDAMENTALS OF CONFINEMENT

Fusion power depends on achieving the physical conditions needed to produce the fusion reactions, i.e., the high temperature to initiate the reaction, and the density and confinement time to yield a net power output.

Confinement is the main scientific issue on which a great deal of plasma physics has been focused. To avoid quenching (cooling) the high-temperature plasma, the plasma cannot be allowed to contact any matter at ordinary temperatures. It must be contained in a sealed vacuum chamber to keep out atmospheric air, and the nuclei of the plasma must not be allowed to touch the chamber walls before they have had sufficient opportunity to collide and fuse. However, at fusion temperatures the nuclei are moving so rapidly that if they were not constrained they would fly to the walls of any chamber of practical size in less than a microsecond.

There are two fundamental ways to treat the confinement problem. These are a) to use magnetic fields to confine the plasma free from contact with the chamber wall long enough for a net release of fusion power; or b) to carry out the processes of heating the fusion fuel so rapidly that the nuclei will react with each other before they can escape to the walls. These two approaches differ from each other profoundly, and define two completely different operating regimes for a fusion reactor as well as for the design of the reactor system. The first is called magnetic confinement, the second is called inertial confinement.

2.1 Magnetic confinement: magnetic field and ion density .

The approach on which most of fusion research has been concentrated is that of magnetic confinement. The plasma associated with the D-T reaction consists of charged particles and neutrons. The neutrons are not affected by the magnetic field, and quickly leave the plasma. The charged particles, on

the other hand, can be constrained by specially shaped magnetic fields to remain within a limited region of the reaction chamber. The strong magnetic fields create a nonmaterial container within which the plasma is held.

A plasma, like any gas, exerts a pressure that arises from the motion of the particles. If the particles are, on the average, sufficiently far apart, the plasma acts like an ideal gas, and the pressure is proportional to the absolute temperature and to the particle density. Figure 3 shows a plot of particle pressure as a function of temperature, based on the ideal gas equation, which may be written in the form

$$p = (n_i + n_e)kT,$$

where $n_i + n_e$ is the total number of particles (ions and electrons) per cm^3 and kT is the kinetic temperature. The line marked "pressure" in Figure 3 is based on assuming $n_i = n_e = 10^{15}$ particles/ cm^3 . For this density the pressure at ordinary room temperatures is less than 10^{-4} atm, while at the critical ignition temperature for the D-T reaction (4 keV) the pressure is about 15 atm.

The maximum plasma pressure that can be confined by a magnetic field depends on the field strength, and since there is a practical limit to the magnetic field strength there is a corresponding limit to the particle pressure of the plasma that can be confined.

The relationship between the magnetic field strength and the effective pressure which the field could exert on the plasma is

$$p + \frac{B^2}{2\mu_0} = \text{constant}.$$

The quantity $B^2/2\mu_0$, which has the units of energy/volume, is the energy density of the magnetic field. However, since energy/volume has the same dimensions are force/area, $B^2/2\mu_0$ may also be regarded as the magnetic pressure of the field. It follows that the sum of the kinetic and magnetic pressures is constant. If the plasma is completely confined by an external

magnetic field of strength B_0 , the pressure at the outside is zero, and the constant is equal to $B_0^2/2\mu_0$, so that

$$p + \frac{B^2}{2\mu_0} = \frac{B_0^2}{2\mu_0}.$$

Note that in the steady state the magnetic field inside a plasma with a finite kinetic pressure must always be less than the external field. It is often convenient to express the kinetic pressure of the particles in a plasma in terms of its ratio to the external magnetic pressure. This dimensionless ratio, β , is given by

$$\beta = \frac{p}{B_0^2/2\mu_0}$$

so that we can also write

$$\beta = 1 - \frac{B^2}{B_0^2}.$$

Since the minimum value of B is zero, the ratio β has a maximum value of unity; this value represents the ideal case of a plasma from which the magnetic field is completely excluded. For $\beta = 1$ the pressure p has the value

$p_{\max} = B_0^2/2\mu_0$, the maximum pressure of a plasma that can be confined,

in a steady state, by an external magnetic field of strength B_0 .

Although stability requirements frequently restrict β in a magnetically confined plasma to values appreciably less than unity, $p_{\max}$ indicates the order of magnitude of the field which is required to confine a plasma in a thermonuclear reactor. For $n = 2 \times 10^{15}$ particles/cm² and $kT = 100$ keV, we find a value for B of about 9T.

A magnetic field of this order of magnitude is not easy to achieve. It cannot, for example, be produced by iron magnets, which saturate at about 2T. Fields of 10T or higher can, however, be produced by current-carrying coils.

The currents have to be very high, and in ordinary materials this leads to large energy losses in the coil proportional to I^2R per unit time. Superconducting coils have no resistance at very low temperatures, and are now being explored for this purpose.

Since the plasma temperatures are extremely high, the other factor that determines the particle pressure, namely the particle density, must be low so that it does not exceed the limit on the plasma pressure. At fusion reaction temperatures of the order of 10^8 °K or higher, the pressure exerted by a gas at atmospheric density would be hundreds of thousands of atmospheres. Consequently the fuel density in a fusion reactor utilizing magnetic confinement must be kept well below atmospheric density in order to keep the pressure exerted on the confining magnetic field (which is ultimately transmitted to the surrounding material structure) within practical limits.

Another factor to be considered in deciding upon the plasma density is heat removal from the fusion reactor. We have seen that the rate at which energy is generated in a fusion reactor increases with the square of the particle density. For energy to be released at a useful rate the density must not be too low, and on the other hand it must not be so high that the fusion energy cannot be removed (and used) as fast as it is released. Figure 5 shows power density values calculated for the D-T reaction as a function of deuteron particle density. (As before the particle densities, n_D and n_T , are taken as equal). The $\overline{\sigma v}$ values at 10 and 100 keV are determined from Figure 2. Also shown in Figure 5 are results for the D-D reaction. If we take into account both the magnetic field strength and the rate of heat removal we see that densities should be in the range of 10^{14} to 10^{16} particles/cm³. We have seen that at the temperatures that must be attained in a deuterium-tritium

fusion reactor, a density of 10^{15} particles/cm³ would represent a pressure of about 15 atm. It is this pressure that must be contained by the magnetic field. The corresponding Lawson criterion confinement times range from about 1 sec to a few hundredths of a sec, depending on the particle density.

For densities of 3×10^{19} deuterons/cm³ Figure 5 shows that at 100 keV a D-D reactor would have a power density greater than 10^9 W/cm³. At the same time the pressure would be about 10^7 atm. Since the confinement time, according to the Lawson criterion, is only a few milliseconds, it is quite clear that the situation would be completely impractical. This would also be true for a D-D reactor at the lower temperature, as well as for a D-T reactor at either temperature.

At the other extreme are the consequences of operating at too low a particle density. When the density is 10^{12} particles/cm³, for example, the D-D power density at 100 keV has fallen to 10^{-5} W/cm³. This value is far too low to be economically interesting. For a D-T reactor the power density would be more than a hundred times greater, but even this would be too small to be of any value.

If the particle densities are in the range 10^{14} to 10^{16} particles/cm³ the fusion power is large. At the lower density it is 1,000 to 300,000 kW/cm³. Since the power density varies as the square of the fuel density it can exceed 10^{12} W/m³ at the higher densities. At the lower power density the power generated would be comparable to that of an ordinary steam power plant and could be handled in a steady state mode. At higher power densities operation would have to be intermittent or in a pulsed mode, resembling that of an internal combustion engine.

2.2 Inertial Confinement.

The basic idea of inertial confinement is that a pellet of a deuterium-tritium mixture is heated very rapidly to fusion temperatures. In this case energy can be produced from plasmas at much higher densities than in a magnetically confined plasma so that the confinement time can be correspondingly less. The physical mechanisms involved in heating the pellet to thermonuclear temperatures for the release of energy are as follows: First energy must be delivered to the pellet. We assume that the energy driver is a laser, but focused beams of high-energy electrons or heavy ions initiated in an accelerator are also being considered. When a pulse of laser energy is absorbed by the pellet a plasma may be formed on the outer surface of the pellet. The resulting pressure causes the outer surface to move outward, and the remaining pellet mass to move rapidly inward. The converging shock pressure compresses and heats the material to the point where the deuterium-tritium reaction can occur. As a consequence of the very large pressures generated by the fusion energy, the pellet explodes. Most of the energy released is again carried by the neutrons. As in the magnetically confined case, the kinetic energy of the neutrons can be converted to heat energy in a moderating region (blanket) surrounding the plasma region. The heated moderator transfers its thermal energy to a working fluid which is then used in a heat engine and electric generator to generate electricity. A new D-T pellet is then introduced and the process is repeated.

Laser-fusion or other beam-driven fusion reactions have to occur so rapidly that the time before the explosion disperses the pellet material provides an adequate confinement time. In order to achieve this inertial confinement the large amounts of energy required to heat the pellet to fusion temperatures must be delivered and absorbed in less than 10^{-9} sec.

2.3 Motion of charged particles in a uniform magnetic field.

We will now consider the principle which underlies the confinement of a plasma by magnetic fields, namely the way in which a magnetic field constrains the motion of charged particles.

Let us consider a uniform magnetic field, that is, one whose magnitude and direction are the same at all points in the region of interest. A particle moving parallel to the field experiences no force. It is only when the particle's velocity $\vec{v}$ has a component $v_{\perp}$ perpendicular to the magnetic field that there is a force on the particle by the field.

The magnetic field is characterized by its flux density (or "induction")

$\vec{B}$. The magnitude of the magnetic force on a particle with charge q is

$$F_m = q v_{\perp} B .$$

The direction of the force is perpendicular to the plane which contains both $\vec{B}$ and $\vec{v}$. This still leaves the possibilities for the direction of the force. The appropriate one is usually determined by some variety of the "right-hand rule." Fig. 6 shows the relation between the various vectors.

Consider now a particle whose velocity has no component v parallel to the field, so that the magnitude of $\vec{v}$ is $v = v_{\perp}$. The magnetic force will remain perpendicular to the velocity, and constant in magnitude. Such a force results in uniform circular motion. In this case the force toward the center (the centripetal force) is the magnetic force, so that

$$q v_{\perp} B = \frac{m v_{\perp}^2}{r} .$$

The circular motion has a radius r and lies in a plane perpendicular to the magnetic field. The radius is given by

$$r = \frac{m v_{\perp}}{qB} .$$

The time for one complete revolution of the particle is called the period T and its reciprocal the frequency f . In the time T the particle travels a distance equal to the circumference $2\pi r$. We can therefore write

$$v_{\perp} T = 2\pi r$$

so that

$$T = \frac{2\pi r}{v_{\perp}}$$

and

$$f = \frac{1}{T} = \frac{v_{\perp}}{2\pi r} = \frac{qB}{2\pi m}$$

The angular velocity of the particle is

$$\omega = \frac{v_{\perp}}{r} = 2\pi f = \frac{qB}{m} .$$

Let us now look at the motion of a particle whose velocity is parallel to the field, so that $v = v_{\parallel}$ and $v_{\perp} = 0$. Since $F = qv_{\perp} B$ there is no force, and the motion continues in a straight line with the same velocity, in accordance with Newton's first law of motion.

In general the velocity will, of course, have both a parallel and a perpendicular component. The motion will then be a superposition of the circular motion in a plane perpendicular to the field, and the straight-line motion with constant velocity $v_{\parallel}$ parallel to the field. The path has the spiral shape of a corkscrew, and is called a helix.

The possible paths of a charged particle in a uniform magnetic field are shown on Fig. 7.

2.4 Magnetic confinement: particle paths.

A uniform magnetic field confines charged particles to some extent: it prevents the particles from escaping in a direction perpendicular to that of the field. There is no barrier, on the other hand, to motion parallel to the field.

One way to close the escape route is to use "magnetic mirrors." These are regions of strongly increasing fields which tend to reflect the particles toward the weaker field. We will show qualitatively how the reflection comes about.

Consider the field configuration shown in Fig. 8a. A positive particle spirals along a uniform field B_x toward a region where the field gets stronger. Where the field gets stronger there is, in addition to the field B_x , a radially inward field, B_r . Now look at these same fields from the left of Fig. 8a, along the x-direction, as on Fig. 8b.

On Fig. 8b a positive particle spirals counterclockwise around B_x . Consider now the additional force f which results from the presence of the radial field B_r . Fig. 8c shows the same fields as Fig. 8b, together with the spiraling particle, and the force f at several points on the particle's path. Application of the right-hand rule shows that f is at each point out of the paper, in the negative x-direction, towards the region of weaker field. A particle which originally travelled in the positive x-direction will tend to be reflected.

There is a second method of confinement which is (at least in essence!) magnificently simple: the region of uniform magnetic field is curved into a doughnut-shape, called a torus. The particles continue to spiral around the field, but instead of escaping along straight field lines they are confined to the torus.

There is now confinement in all directions, but it is far from perfect. Some particles will have orbits which carry them out of the field region. The particles will also make collisions with each other which will change their velocities and their paths. This is the process of diffusion, and it will lead to a loss of particles from the confinement region.

The loss of particles becomes smaller as the tube diameter increases. In other words, as the diameter gets larger so does the confinement time.

2.5 The Tokamak.

A "tokamak" is a device based on the doughnut-shaped arrangement. The word is a Russian acronym for "toroidal chamber in a magnet field." There is a field around the torus as described in the previous section. That, however, is only the beginning!

The plasma needs to be heated. This is done, in part, by making the plasma tube itself the single-turn secondary of a transformer. The transformer action sets the plasma into motion around the torus as a plasma current. The collisions which the plasma particles make cause electrical resistance, just as for an ordinary current in a wire, and this leads to heating ("ohmic heating"). Heat may also be supplied by beams of particles directed at the plasma.

There is a further small field along the axis of the torus to maintain the position of the plasma current, as well as additional correction fields. Needless to say, an actual system has a wide variety of complexities.

A device of the sort that we have described should be able to provide the necessary temperature and confinement time for a self-sustaining thermonuclear reactor. Present systems are within an order of magnitude of the Lawson criterion. Larger devices, with a consequent increase in confinement time, are now being built, and are expected to allow self-sustaining fusion reactions to take place.

2.6 Fusion reactor design.

At a time when no self-sustaining fusion reaction has yet been produced it may seem premature to discuss the details of a possible fusion reactor system. Nevertheless quite extensive design studies have been made, both to investigate the kinds and sizes of the components which are likely to be required, and to get an idea of the cost and economic viability of such a system.

Fig. 10 shows the essential elements of one possible configuration. Deuterium and tritium are introduced into the evacuated toroidal chamber. Heating causes the atoms to be ionized so as to form a plasma. The plasma is further heated to the ignition temperature at which thermonuclear reactions take place with sufficient energy output to maintain the temperature. Superconducting magnetic field coils maintain the confinement conditions for some seconds, at ion densities of the order of 10^{14} per cm^3 , so as to exceed the Lawson criterion.

The neutrons fly to the wall of the chamber, which contains lithium to produce tritium and, in this design, also graphite to slow the neutrons down. Since the neutrons carry 80% of the energy which is produced in the D-T reaction their absorption in the wall also leads to the main heat production.

The heat is removed by a gas or fluid. The size of the chamber is determined primarily by the rate at which the absorbed heat can be carried away.

The hot gas or fluid takes the heat which is produced to a heat exchanger where it produces steam for a conventional turbine which runs an electric generator.

The design parameters for this system are shown in the following table.

TABLE 1
REPRESENTATIVE OPERATING CHARACTERISTICS FOR TOKAMAK REACTORS

Parameter	Value
Plasma radius	2 to 5 m
Major radius of torus	10 to 13 m
Plasma volume	830 to 6400 m ³
Ion density	0.5 to 2 x 10 ¹⁴ /cm ³
Ion temperature	10 to 30 keV
Maximum toroidal field	80 to 160 kG
Axial current	8 to 21 MA
Fusion power density	0.7 to 2.0 MW/m ³
Fusion power output	1000 to 5000 MW
Confinement time	several sec to tens of sec
Burn time	hundreds to thousands of sec
Purge and reload time	30 to 60 sec
Auxiliary heating power	10 to 100 ME
Fuel feed rate	2 to 4 x 10 ²² /sec
Neutron wall-loading	1 to 4 MW/m ²

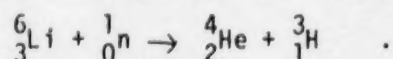
An inertial confinement system would not have the magnetic field coils. Instead there would be a chamber into which pellets containing the deuterium and tritium would be introduced. Bombardment by lasers or by particle beams would produce the heating. The rest of the components would be similar, in principle, to those in a magnetic confinement system.

SEGMENT 3. SAFETY AND POLLUTION OF FUSION REACTORS

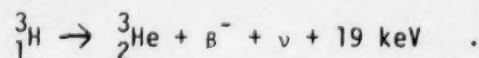
3.1.1. Tritium.

It is conceivable that fusion reactors will one day be so highly developed that the D-D reaction, with its higher ignition temperature and much higher Lawson criterion, can be used. In the meantime the D-T reaction seems to be the only one with a reasonable likelihood of success, and we must therefore consider the tritium hazards.

The tritium is to be made through the reaction



It is radioactive and decays with a half-life of 12.3 y according to the equation



The neutrinos, as always, escape without any interaction. The beta rays have a lower energy than those from almost all other beta-ray emitters. In ordinary air they are stopped after travelling less than 1 cm. Except under the most extraordinary circumstances the external radiation hazard is therefore likely to be negligible, and the main danger is that of getting some tritium into the body.

Since the chemical properties of tritium are like those of all hydrogen it will distribute itself throughout the body, changing place with ordinary hydrogen wherever it occurs. It will therefore also be eliminated from the body with the ordinary hydrogen. We can define a "biological half-life," equal to the time for half of the tritium in the body to be eliminated, which turns out to be several days.

This behavior is quite different from that of some other radioactive materials. Radium and strontium, for example, are chemically similar to calcium, and therefore concentrate in the bones, where they tend to remain without much chance for natural elimination.

3.1.2 Neutron activation.

The neutrons which are produced in the D-T reaction, and which are not absorbed by the lithium, will, for the most part, be absorbed by initiating other nuclear reactions in the surrounding material. Some of these reactions will have radioactive products.

The materials will have to be chosen so as to minimize this induced radioactivity, but there will be a certain amount of radioactivity to be controlled and to be disposed of.

3.1.3 Radiation damage.

The neutron bombardment of the wall material will be intense. Such bombardment causes the atoms of the material to be dislodged from their normal positions. Consequently the crystalline arrangement is altered, and the strength of the material is affected. The details are complicated, and our knowledge of the effects to be expected is very limited.

3.1.4 Accidents.

The accidents which one can envision for a fusion reactor are not very different from those which can occur in other systems which contain or release a large amount of energy. There is no mechanism for a nuclear explosion or any kind of continuing nuclear chain reaction.

Normal operation will presumably represent conditions close to the optimum in containment characteristics and power production. Any change or malfunction can then only lead to a lowering of the power level.

The most serious kind of accident is probably one which results from damage to the wall of the reactor chamber. It could, for example, fail as a result of weakening from radiation damage. It could overheat and be destroyed if the cooling system failed. In both cases there would be great damage to the reactor. Danger beyond the immediate vicinity would presumably come only from any tritium which escapes.

3.1.5 Comparison with fission-reactor hazards.

Commercial fission reactors have been in operation for more than two decades, so that we now have considerable experience with them. There continues to be a great deal of public discussion and controversy about them.

Fission reactors are subject to several hazards which are quite different from those to be expected from fusion reactors. One is that vastly more, and more dangerous, radioactive material is produced in a fission reactor. There are, first, the fission products, and, second, the transuranic elements, principally plutonium.

The amount of radioactivity which is produced is so great that right after shut-down a fission reactor continues to produce heat at about 7 per cent of its full power. As a result of the huge amount of radioactivity the dangers from accidental release and the problems of waste disposal are much greater than those which can be envisioned for a fusion reactor.

Present commercial reactors are slow-neutron reactors, in contrast to the "breeder reactors" which are designed to produce plutonium to be used as fuel. (Slow-neutron reactors also produce plutonium, but in much smaller quantities). Plutonium is not only a very hazardous material, but it can be used to make nuclear ("atomic") bombs.

A breeder-reactor accident is potentially very dangerous because of the high energy density of the reactor, and because of the large amounts of plutonium which it contains.

3.2 Problems of scale and cost.

Whether a particular technically feasible system of power generation is actually put into operation ultimately depends on its cost in competition with other possible systems. The term "cost" should here be understood in its widest possible sense. We may or may not, for instance, count the price of removing atmospheric pollution when we consider the cost of running a coal-burning plant. We may or may not be able to measure, as part of the cost, the suffering endured by the population when the pollution is not removed.

The obvious cost advantage of a fusion system is that the fuel is cheap. The obvious disadvantage is that a large and complicated plant is required. Present estimates are that electrical power from a fusion reactor will be more expensive than that which we use today by a factor of between two and three. But present systems may become more expensive as coal and uranium resources dwindle.

It may be useful to mention solar power, at least briefly. It, like fusion, has no fuel problem, but the installation cost is large. At present it looks to be even more expensive than the projected fusion power, but research is carried on at an increasing rate, and extrapolations into the future are notoriously unreliable.

Solar power is different from fusion and fission power in that it is "dilute." That is, it does not depend on a huge installation of some minimum size, with a large power density that carries with it the potential for serious accidents. On the other hand its very diluteness is a source of difficulty. The area required to collect enough energy from the sun to equal that generated in one of our current power plants is very large.

We do not know how much energy the world will use in the future. We do not know what form the energy will take. No form of energy generation is

without its problems, starting with man's original chain reaction, the fire produced by rubbing together two sticks.

Decisions will have to be made on the way energy is used, and on the way in which it is produced. We have tried in this module to provide the background for greater understanding of some of the issues and problems which will have to be faced.

3.3 Problem set 3

1. In the usual form of the ideal gas equation, $pV = nRT$, n is the number of moles, and R is the gas constant. $R = kN_A$, where k is Boltzmann's constant and N_A is Avogadro's number.

Now let n_T be the total number of particles per unit volume in the gas. Show that the ideal gas law can be written $p = n_T kT$. (See p. 20).

2. For $n_i = n_e = 10^{15} \text{ cm}^{-3}$ (as on p. 20), calculate the pressure at room temperature and at $kT = 4 \text{ keV}$. (Use MKS units to get p in N/m^2 , and convert to atmospheres, using $1 \text{ atm.} \approx 10^5 \text{ N/m}^2$).

3. Calculate n_T in cm^{-3} at room temperature, from the ideal gas law (see problem 1), and compare with the value given on p. 23.

4. Calculate p in atmospheres for $n_T = 3 \times 10^{19} \text{ cm}^{-3}$ and $kT = 100 \text{ keV}$, and compare to the value given on p. 24.

5. For $\beta = 1$, $n = 2 \times 10^{15} \text{ cm}^{-3}$, $kT = 100 \text{ keV}$, calculate the magnetic field B . (p. 22).

6. A deuteron has a kinetic energy of 1 keV . What is its speed?

7. A deuteron with a kinetic energy of 1 keV moves in a circle under the action of a uniform magnetic field with B equal to 3T . What is the radius of the path?

8. A deuteron with a kinetic energy of 1 keV moves at an angle of 45° to a uniform magnetic field with $B = 3\text{T}$.

(i) What is the component $v_{||}$ of the velocity, parallel to B ?

(ii) What is the component $v_{\perp}$ of the velocity, perpendicular to B ?

(iii) What is the radius of the circular motion corresponding to $v_{\perp}$? (We are really asking for the radius of the helical motion!)

(iv) What is the time T for one complete revolution?

(v) How far does the particle travel in a direction parallel to B (as a result of the component $v_{||}$) in the time T ?

9. Consider a region of uniform magnetic field B_x as on Fig. 8, but bounded on the left by a region of increasing field. Draw figures like Fig. 8, a, b and c for this case, and show that the forces on a particle spiraling toward the increasing field are such as to reflect the particle toward the region of weaker field.

3.4 Projects, questions, and suggestions for further reading and research.

1. (a) Describe the fate of a neutron in a fusion reactor: how is it created, what happens to its kinetic energy, what are the various ways in which it may eventually disappear?

(b) Describe the fate of a neutron in a fission reactor: how is it created, what happens to its kinetic energy, what are the various ways in which it may eventually disappear?

2. Describe the fate of an alpha-particle in a fusion reactor: how is it created, what happens to it and to its kinetic energy?

3. Describe the role of tritium in a fusion reactor. What mass of tritium is used each day in a reactor which produces 3000 megawatts of heat power? Describe the production of tritium, its radioactive decay, and the hazards associated with its radioactivity.

4. Discuss, as quantitatively as you can, the problems associated with neutron-activation and radiation damage in a fusion reactor.

5. Compare the hazards resulting from radioactivity for a fusion reactor and a fission reactor.

6. Compare the possible accident hazards for a fusion reactor and a fission reactor.

7. Discuss the environmental hazards of a fusion reactor and compare them to those for a fission reactor.

8. Calculate, compare, and discuss the following:

(a) What is the raw material necessary for a year's operation of a fusion reactor using the D-T reaction, producing 1000 megawatts of electricity at an efficiency of 30%?

(b) Do the same for a fission reactor using ^{235}U .

(c) What is the area of solar collectors which is required for the same electrical power, if 10% of the solar energy is converted to electricity,

and if the average intensity of solar radiation is 14% of the "solar constant." (The solar constant is the intensity of solar radiation on a surface normal to the sun's rays above the earth's atmosphere. It is 1353 W/m^2).

9. Study the article "The prospects of fusion power" in Scientific American for Feb., 1971. Discuss the changes which have taken place since that time from the following points of view:

- (a) technical progress toward fusion power;
- (b) changes in forecasts for energy consumption;
- (c) technical progress toward alternative energy sources;
- (d) changes affecting the use of fossil fuels.

Table 2

speed of light	$c = 3.00 \times 10^8 \text{ m/s}$
electronic charge	$e = 1.60 \times 10^{-19} \text{ C}$
Planck's constant	$h = 6.63 \times 10^{-34} \text{ J}_S$
Boltzmann's constant	$k = 1.38 \times 10^{-23} \text{ J/K}$
Universal gas constant	$R = 8.31 \text{ J/K gram-atomic mass}$
Avogadro's number	$N_A = 6.02 \times 10^{23} \text{ atoms/gram-atomic mass}$
Coulomb force constant	$K = 1/4\pi\epsilon_0 = 9.00 \times 10^9 \text{ N m}^2/\text{C}^2$
magnetic force constant	$\mu_0/4\pi = 10^{-7} \text{ N/A}^2$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \quad (\text{electron volt to joule})$$

$$1 \text{ u} = 931.5 \text{ MeV} \quad (\text{unified atomic mass unit to million electron volt})$$

$$1 \text{ T} = 10^4 \text{ G} \quad (\text{tesla to gauss})$$

Table 3

masses of elementary particles:

$$m_e = 9.11 \times 10^{-31} \text{ kg} = 5.49 \times 10^{-4} \text{ u} = 0.511 \text{ MeV}$$

$$m_p = 1.6725 \times 10^{-27} \text{ kg} = 1.00728 \text{ u} = 938.3 \text{ MeV}$$

$$m_n = 1.6748 \times 10^{-27} \text{ kg} = 1.00866 \text{ u} = 939.6 \text{ MeV}$$

atomic masses

(these are the masses of the neutral atoms in unified atomic mass units)

${}^1_0\text{n}$	1.00866	${}^3_2\text{He}$	3.01603
${}^1_1\text{H}$	1.00783	${}^4_2\text{He}$	4.00260
${}^2_1\text{H}$	2.01410	${}^6_3\text{Li}$	6.01513
${}^3_1\text{H}$	3.01605	${}^7_3\text{Li}$	7.01600

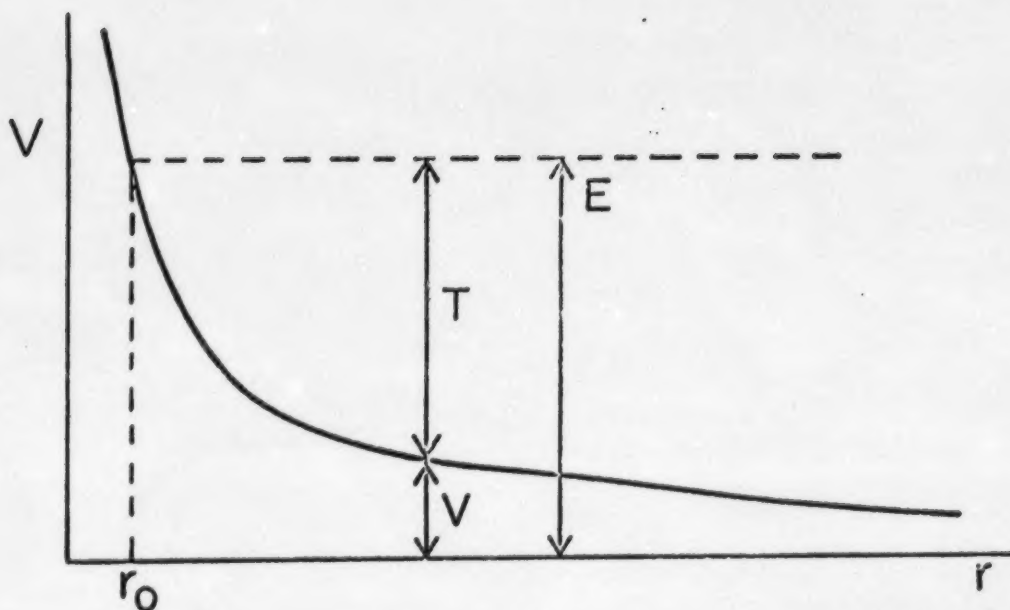


Fig. 1. The electrostatic potential energy of two nuclei as a function of the distance between them.

The solid line represents the electrostatic potential energy $V = K Z_1 Z_2 e^2 / r$. The horizontal line represents the constant total energy E . At any point along the horizontal axis the distance up to the solid line is the potential energy V , the rest, up to the value E , is the kinetic energy T . The closest distance of approach, r_0 , corresponds to the point where all the energy is potential energy. When the particles are very far from each other, the potential energy is negligible, and the kinetic energy is equal to the total energy.

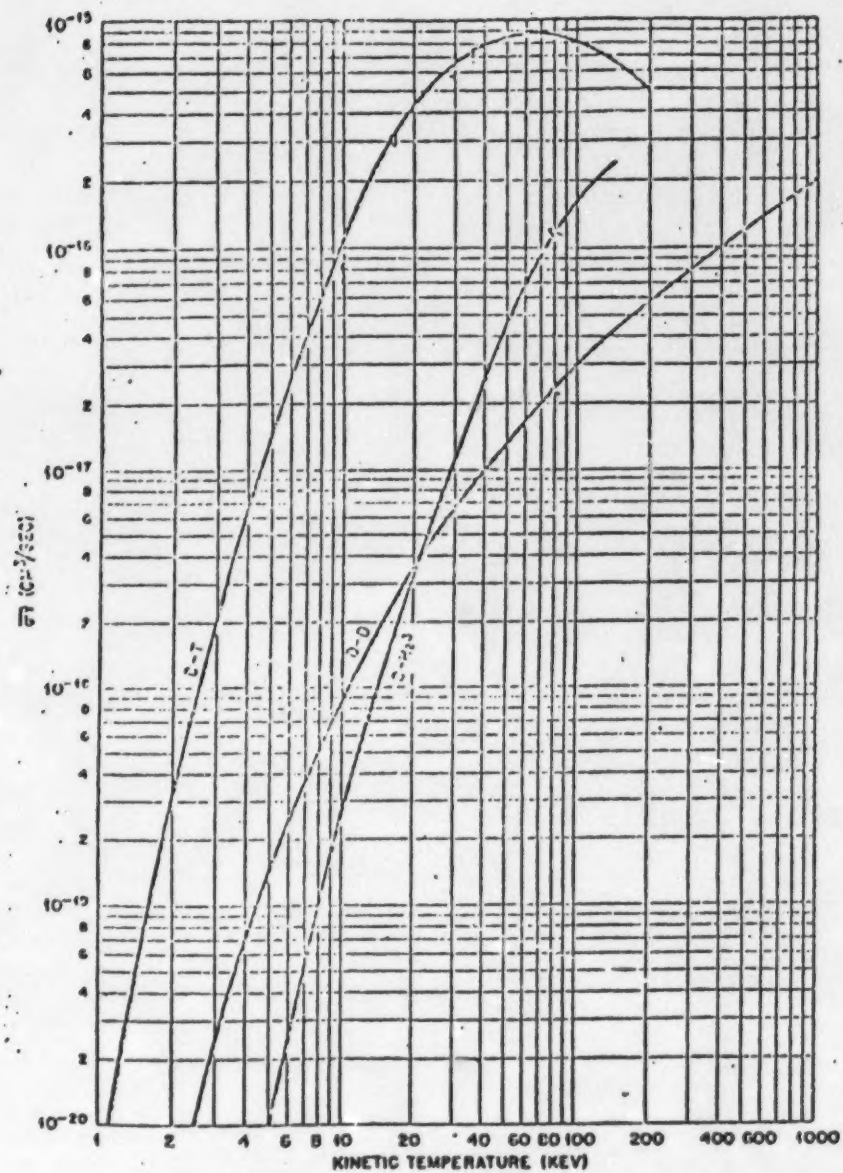


Fig. 2. Values of $\overline{\sigma v}$ for the D-T, D-D (total), and D- ^3He reactions.

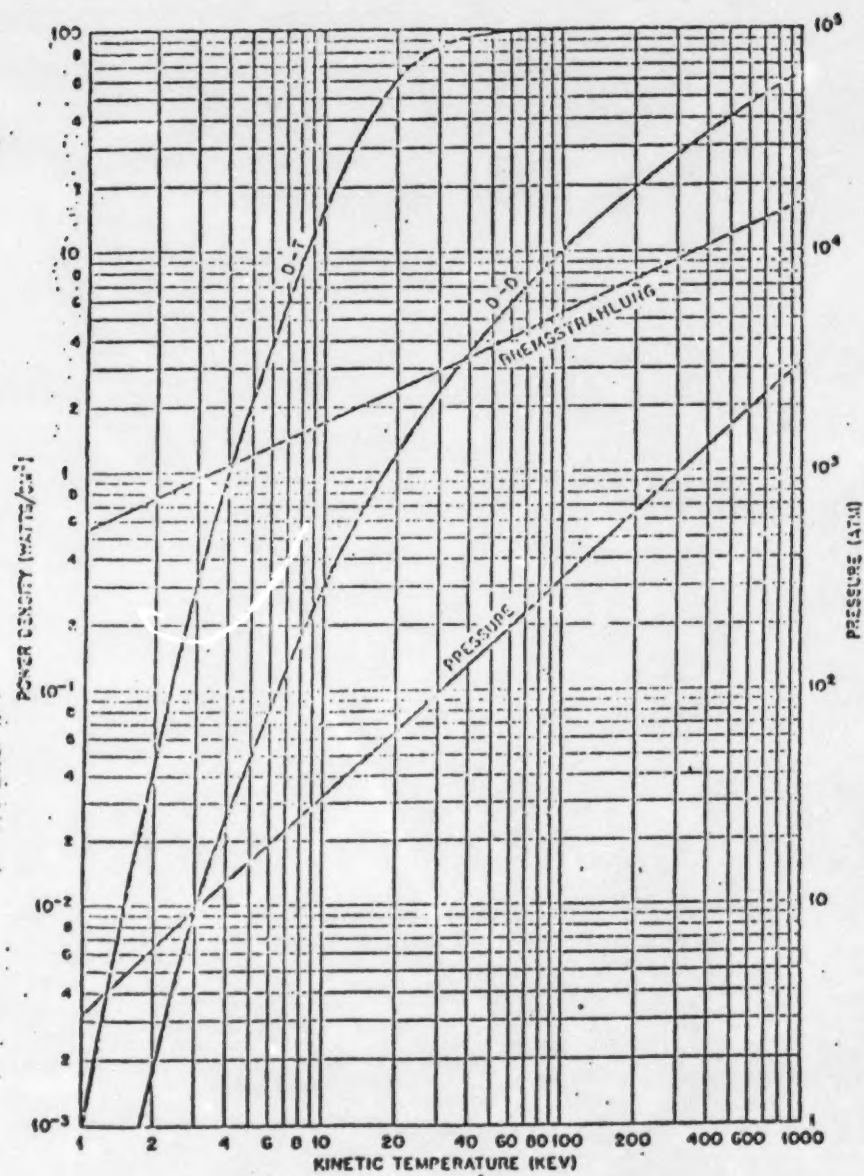


Fig. 3.

Power densities produced by the D-T and D-D reactions; Bremsstrahlung power density and pressure.

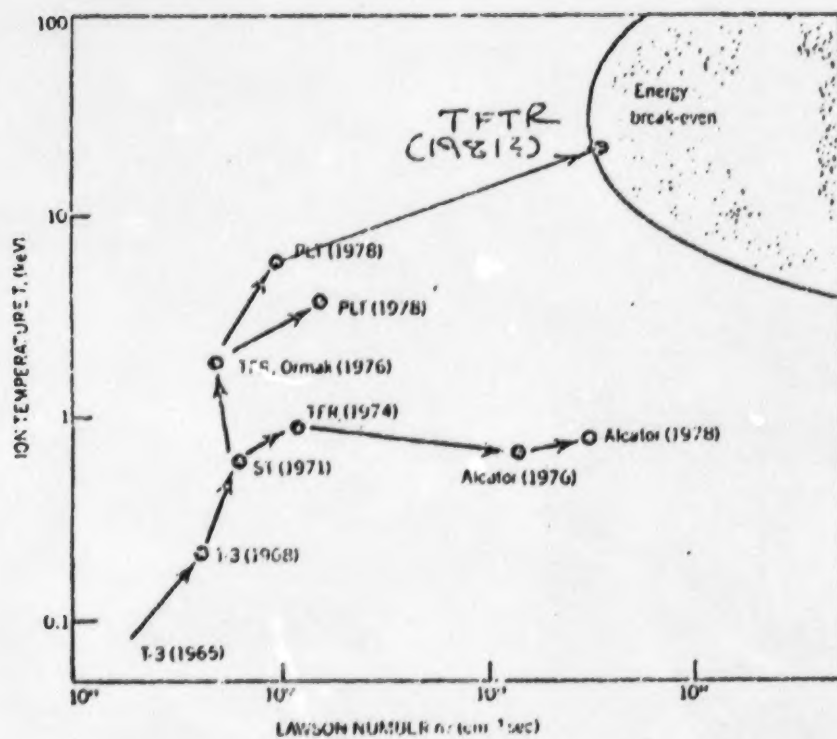


Fig. 4. Ion temperatures and Lawson numbers ($n\tau$) for various fusion devices.

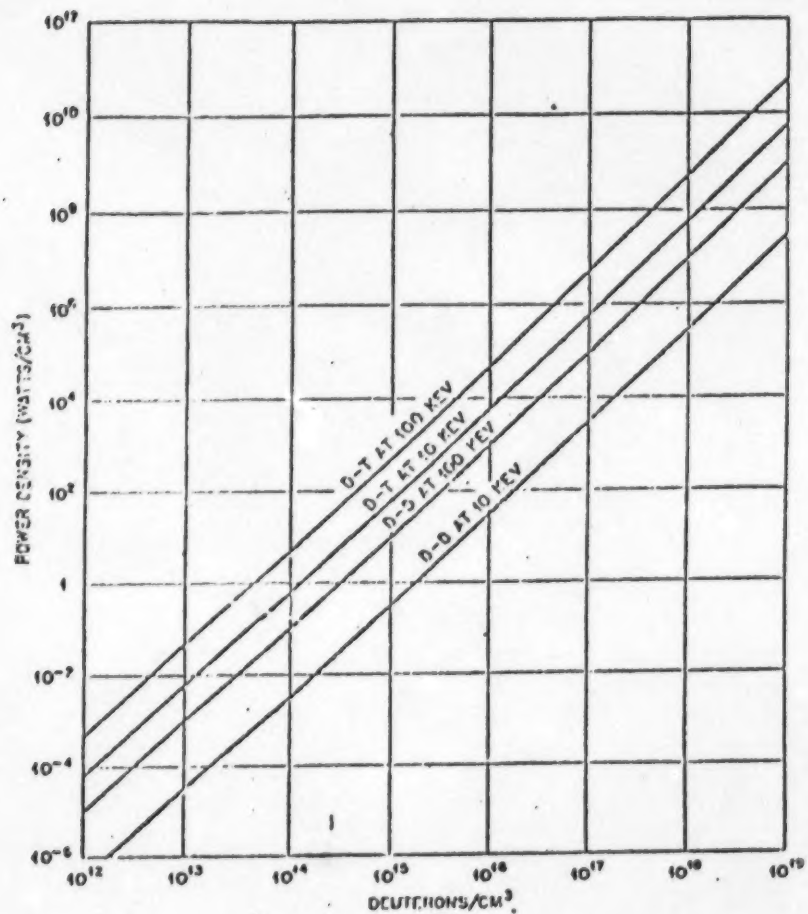


Fig. 5. Power densities for the D-T and the D-D (total) reactions as a function of deuteron density.

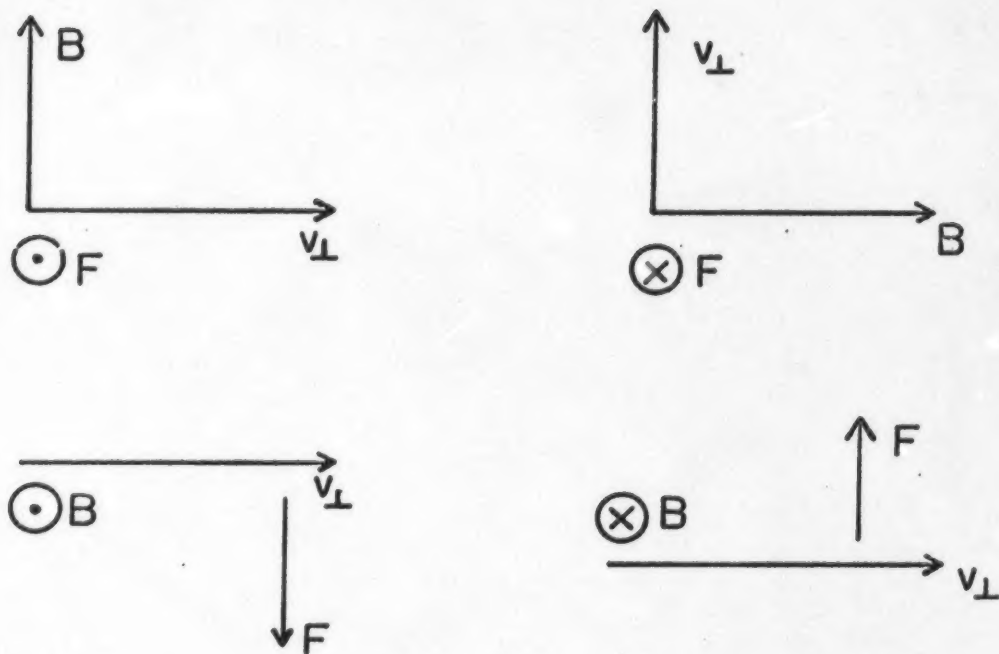


Fig. 6.

Force on a positive particle in a magnetic field. v is the component of the velocity perpendicular to the field. A vector into the paper is represented by a cross, a vector out of the paper by a dot. Right-hand rule: stretch the fingers of the right hand in the direction of v . Now bend them so that they point in the direction of B . The thumb then points in the direction of the force on a positive particle.

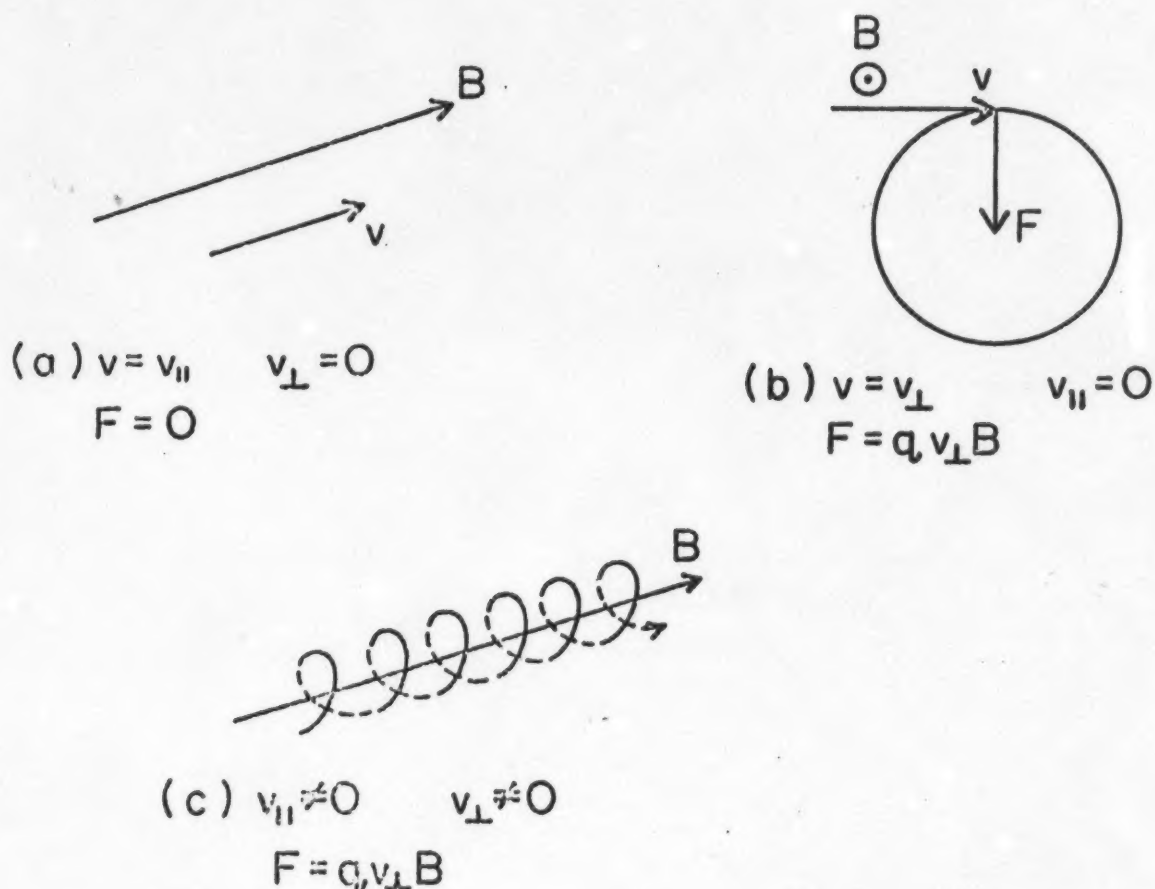


Fig. 7. The possible paths of a particle in a uniform magnetic field.

(a) The velocity is parallel to the field. There is no force and the particle continues in a straight line with constant velocity.

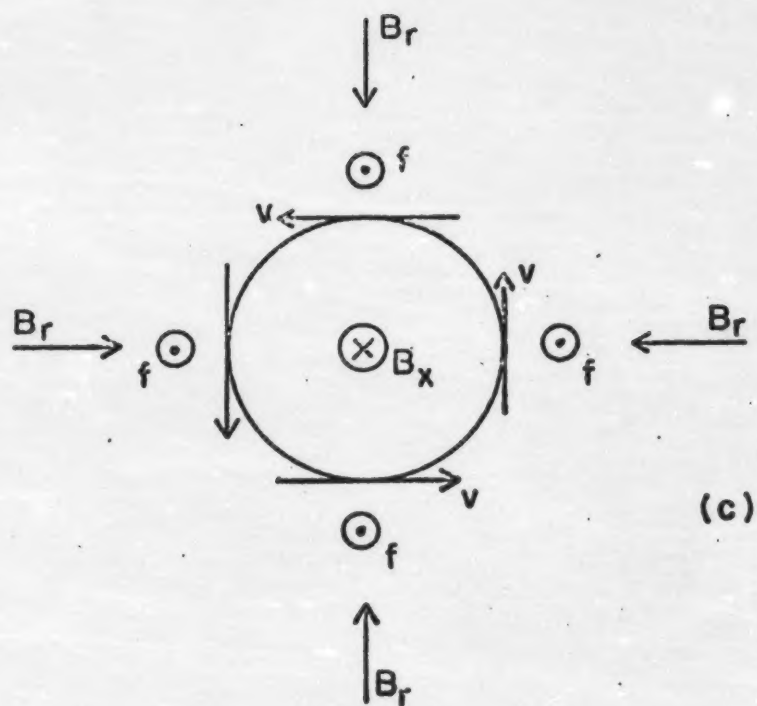
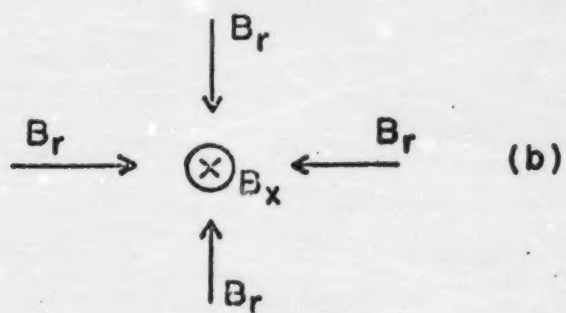
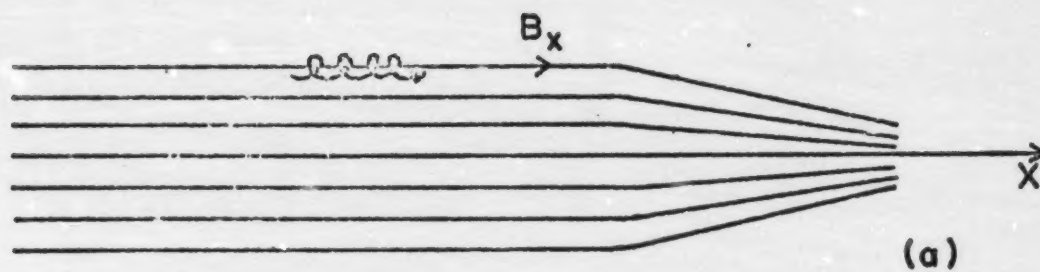
(b) The velocity is perpendicular to the field. The magnitude of the force is qvB , in a direction at right angles to both the velocity and the field. The magnitude of the velocity remains constant and the particle moves in a circle, according to $qvB = mv^2/r$.

(c) If the velocity has a component $v_{||}$ parallel to the field, and a component $v_{\perp}$ perpendicular to the field, the motion is a superposition of the straight-line motion of part (a) and the circular motion of part (b). The component $v_{||}$ remains unchanged; the component $v_{\perp}$ remains constant in magnitude, but not in direction. The path is a helix whose axis is parallel to the field. Note that in no case does the particle's energy change as a result of the magnetic force.

The diagrams are drawn for positive particles. The force on a negative particle would be in the opposite direction so that the direction of the circulation in parts (b), and (c) would be in the opposite sense.

- Fig. 8. Action of a magnetic mirror.
- (a) A particle spirals along the uniform field B_x toward a region where the field increases.
 - (b) In the region of increasing field the vector B has two components: the x-component B_x , and a component B_r radially inward. This diagram shows both components. The x-direction is into the paper.
 - (c) A positive particle moves in a circle, counter-clockwise, in the plane of the paper. Its velocity is tangential to the circle. The radial field B_r gives rise to a force f out of the plane of the paper, toward the region of the weaker field.

(Figure on page 52.)



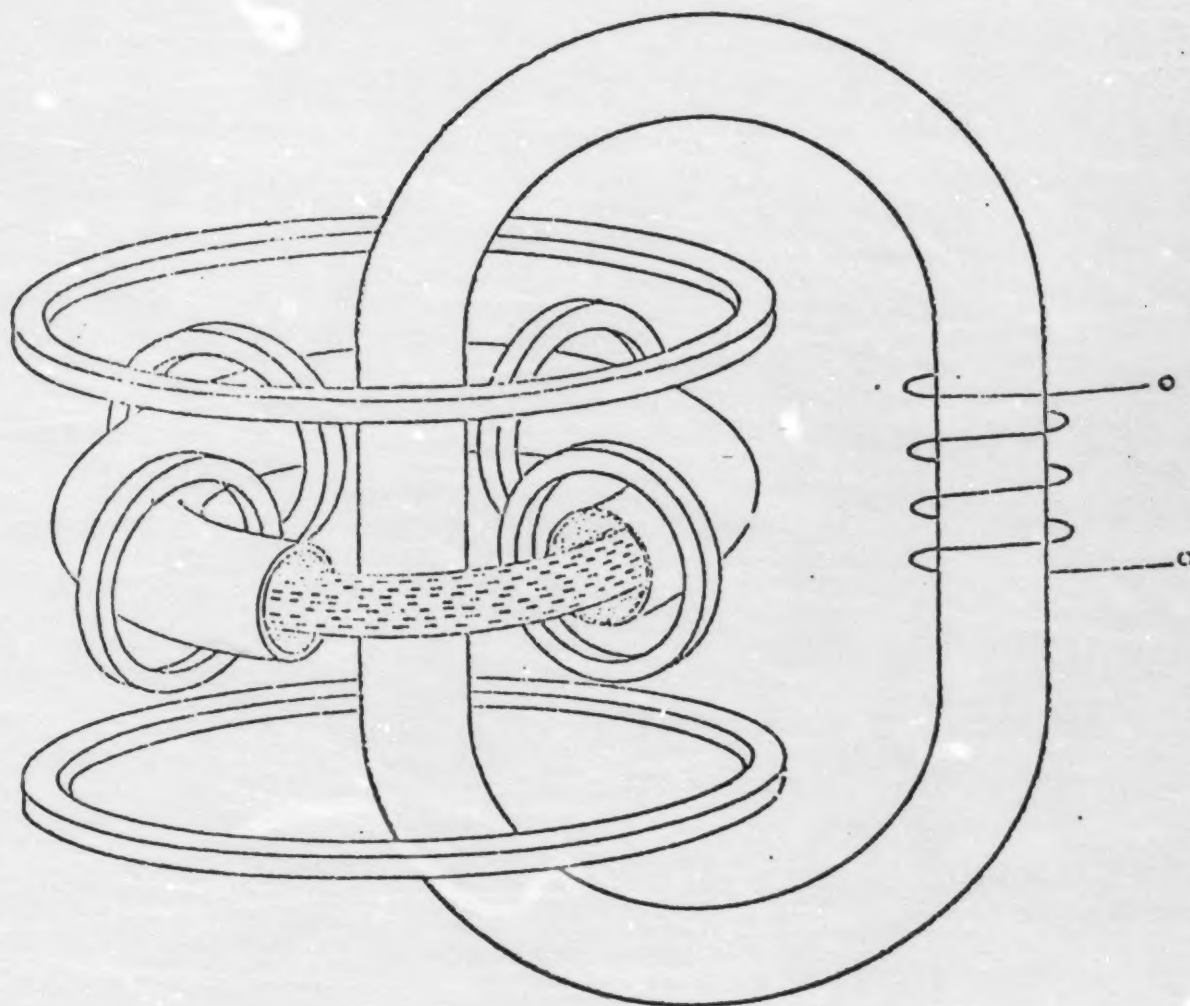


Fig. 9.

Some of the basic components of a tokamak.

The figure shows the plasma in its toroidal shell, the coils for the toroidal field, the coils for the vertical field, and the transformer for the ohmic heating of the plasma.

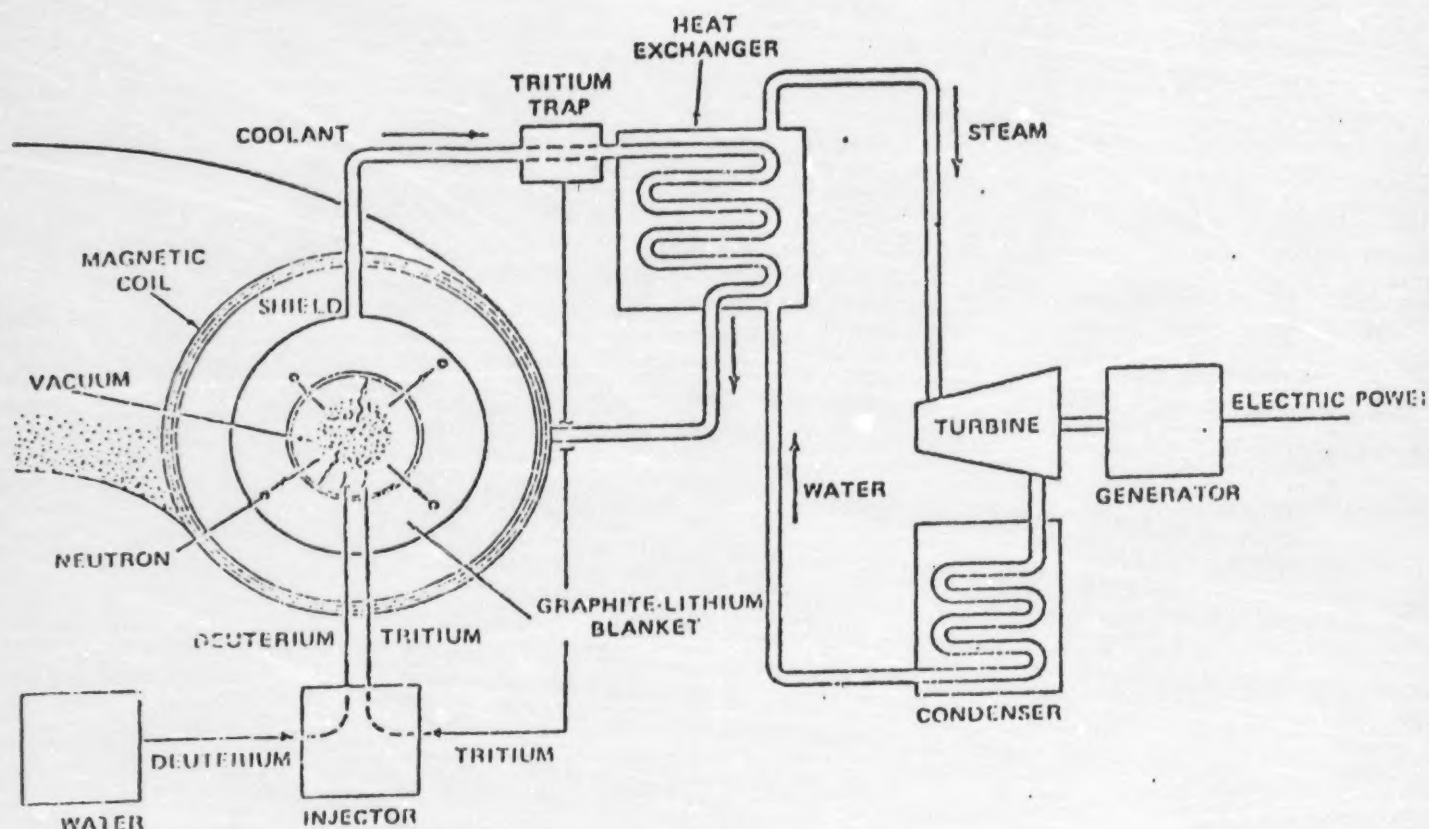


Fig. 10. Design for a fusion reactor. The deuterium and tritium are injected into the plasma inside the toroidal shell where the nuclear reactions take place. The alpha-particles remain in the plasma and contribute to its energy. The neutrons fly to the wall. There they cause further nuclear reactions, including those which produce ^3He from lithium. The heat is carried away by the coolant, and changed, in part, to electrical energy in a conventional power plant.